Chapel in the White Tower, London

Discrepancies in medieval architecture, careless or deliberate?

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There is a difference between tolerances, mistakes and deliberate distortions. Tolerances are those small errors in cutting and erection that occur inevitably because none of us are perfect. Mistakes, well, that is being human. But deliberate distortions? The chapel in the white tower in London is an excellent demonstration of how important it is to understand the difference.

As is well known most medieval buildings suffer from bent axes, variable bay dimensions and other apparent errors. This is somewhat puzzling when we consider the otherwise excellent craftsmanship in cutting and erection. It seems inconsistent that masons who could follow the template to within a millimetre and who could assemble stones to almost the same accuracy could also make mistakes in setting out which are at times in the order of hundreds of millimetres.

There is good reason to think they are not all errors, but that many were deliberately introduced. Today we are so carefully trained to maintain modular accuracy that any deviation from this perfection seem like carelessness. Yet in the other arts such irregularities are well-known, and understood as part of the structure of the work. In poetry,[1] music,[2] and in some pattern-making[3] the phenomena of tempering, 'spezzaturo',[4] and Tempo Rubato are well known. These are deliberate breaks in symmetry, changes of pace or rhythm which enhance the main theme and add a touch of the unexpected.

Nevertheless, in architecture the persistent use of distortions to overcome problems in geometry has been referred to as 'accidental', or as 'miscalculation'.[5] Similarly, discrepancies in music have been attributed to 'the carelessness or, at best, the indifference of many composers towards precise rhythmic notation,'[6] so that scores have been corrected in much the same way as restorers have corrected the not dissimilar inconsistencies found in medieval churches. In poetry, the numeric structure of which is now being analysed in detail, distortions of a similar kind have been found. They are referred to as 'textual defects or irregularities' or the result of 'confusion' by a copyist.[7]

There is a growing appreciation of the role that tempering plays in all the arts.[8] In music the 'uniformitarian' assumption of classical musicology is being reassessed, and it is being argued that 'a paradigm of inconsistency' can be applied to virtually all the notated variables of music.[9] In the introduction to Beethoven's sonatas we are warned that when he "shows a discrepancy between parallel passages … the editor should never adjust it except where one of the readings is unintelligible … With Beethoven nothing is more unsafe than to correct him by aiming at uniformity … His art is not tidy".[10]

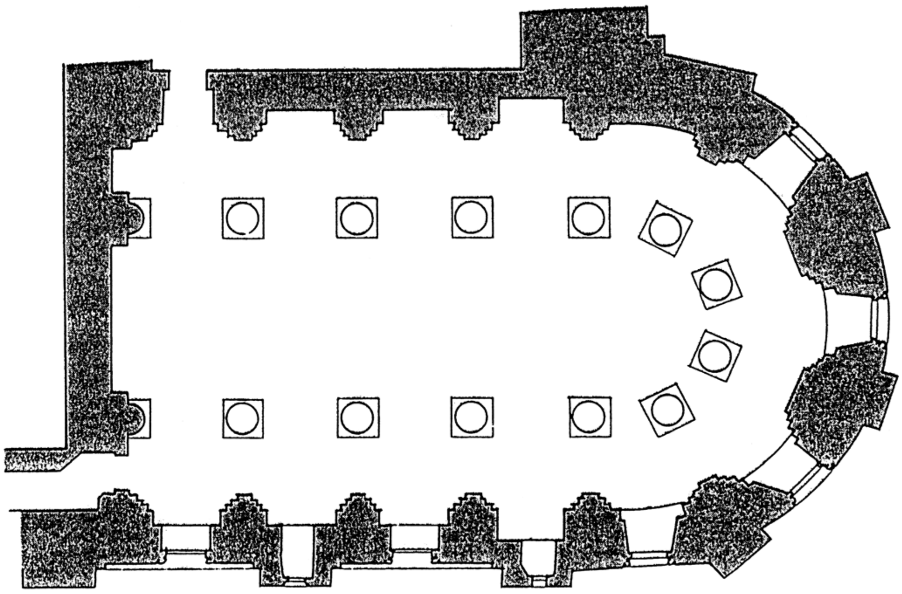
Medieval architecture is not tidy either, and the chapel in the White Tower shows it was evolved with the utmost geometric care, Fig. 32-1. As geometry reasoning is the source of any discrepancies, the reasoning behind them may illuminate a number of important concepts held by medieval architects. The more common discrepancies seem to be:

- The bending of the main axis, found in over half the medieval churches in any country. It is usually misplaced to the south, and at times the bend is clearly visible. One only has to examine a book of church plans to see how common it is.
- The confusing use of different centres to locate the walls of the apse, the piers of the apse and the flanking piers of the nave. One result is that the radial axes through the ambulatory piers and wall pilasters seldom line up with the central boss, and the width of the vaults of the ambulatory increases towards the east.
- The avoidance of the square in most aisle bays. The inequalities of the sides may not be large, and could have been squared up if desired, and would have made it easier to set out the vaults.
- The thickness of the walls differ, those along the sides often being considerably thinner than those at the ends. Sometimes walls are bowed, bent or stepped.

We can see why some of these discrepancies were made. The answers may not be applicable to all buildings, as the way these matters were handled seems to have depended on the whims or training of the master in charge, but the logic behind them shows that these discrepancies originated as deliberate adjustments to the plan by the master during the design process. The consistency of their logic, and the care with which it was implemented, show that the adjustments were intended, and not mistakes.

Discrepancies in many buildings stem from the discontinuous contractual system and the loose supervision that resulted.[11] But in one eleventh century building the urgency of the work, the ample funds and the architectural evidence suggests that the entire structure was the work of one master, and so would be ideal for this purpose – the Chapel of St John in the White Tower, London, started by William the Conqueror around 1087.[12]

Simple as the plan appears, it was designed in a surprisingly intricate manner. It will illustrate a reasoning behind each of these adjustments, and may help us understand similar discrepancies in larger buildings.

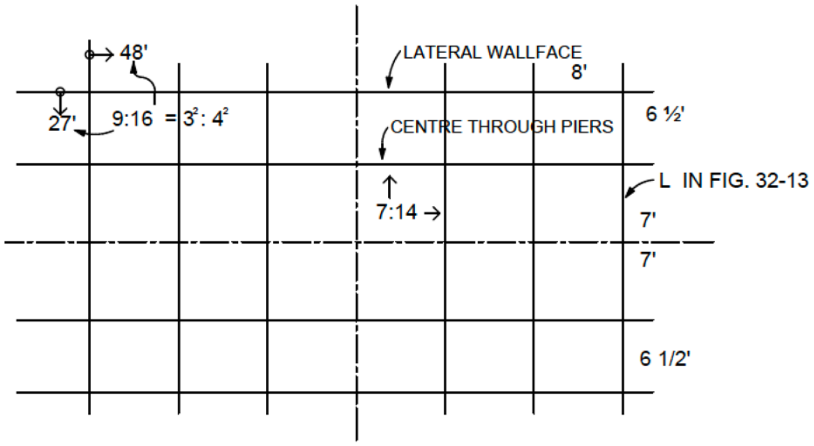


To establish the geometry, I measured the chapel to a greater accuracy than the expected cutting or erectional tolerances. From them I prepared as-built drawings to a scale that would show up any discrepancies of measurements.[13] All dimensions were written onto the drawing, and the building tolerances calculated.

The ratios between the parts were investigated with slide rule and calculator, until every detail, every wall and shaft could be determined from a series of coherent, interconnected geometric steps that should be consistent, usually mixing whole numbers with irrationals. Once the geometry emerges it was checked by calculation, from which I prepared an ‘idealised’ drawing of the geometry

The tolerances were simple to calculate. With one exception the average bay measured 2,903 mm, ranging from a maximum of 2,910 to a minimum of 2,895 mm: a scatter of –8 to +7 mm. The first bay on the south is the exception, being +65 mm from the mean. Excluding it for reasons that will appear, the tolerances for the rest are as good as we would expect today. The bases under the piers are cut from single stones. Their width averaged 936 mm with a scatter of –1 to +3 mm. This is reasonably good cutting, and not atypical of the best medieval buildings. Thus, tolerances were ±3 mm in cutting and ±10 mm in placement. Any acceptable geometry would therefore have to fit within these limits.

The next step is to determine the 'ideal' dimensions. For example, the average for the span and bay are 5,085 and 2,903 mm. The slide rule shows they relate as 7:4. By adding the average span to the average bay, and dividing by 7+4, we get the 'ideal' dimensions of 5,083 and 2,904 mm, which are closer to the averages than the tolerances of ±10 mm. A quite remarkable convergence.

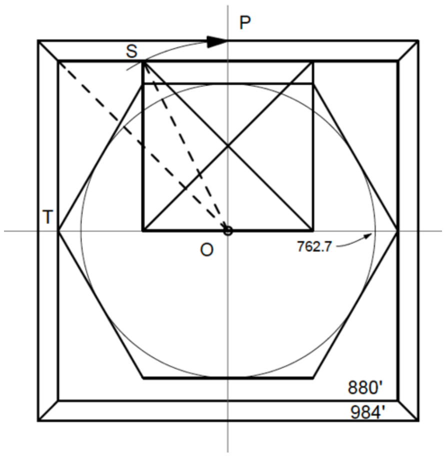


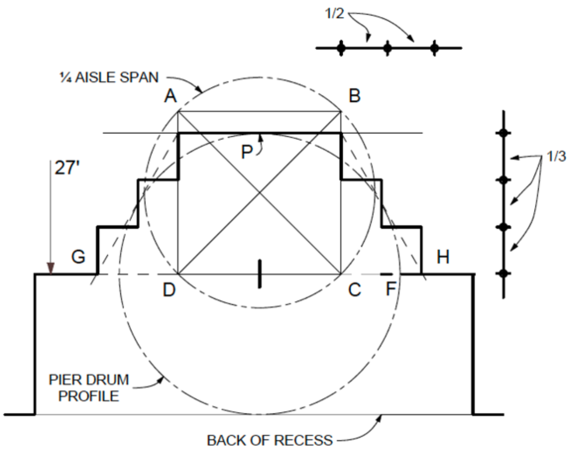
Bay and aisle dimensions

The four bays of the nave, each in the ratio of 7:4, combined to form a 7:16 rectangle. This established the core of the interior. To this was added a further two bays, which as we shall see located the pilasters to the curved wall of the apse.

A second rectangle was placed around these six bays in the related ratio of 9:16. It forms the pleasing arrangement of $3^2.2^4$ [above, left]. Both rectangles pass through the axes in both directions: being those of the piers in the first, and of the pilasters in the second. As they coincide at the western bay, the apse occupies the space left between them.

Around the intersection of these axes circles were drawn, at 3/20th of the span. The templates for the piers and the pilasters were taken from these circles, including the recesses in the walls between the pilasters. These steps are described below.





The width of the aisle is the result of this process. They are narrow, and the vaults cramped. Since the aisles were 'derivative' their proportions were not intrinsic, but came from the geometry for other parts of the building. If only 180 mm were added to the back of the recesses, the aisle bays could have been made square. Little enough to show that the 7:16 ratio in the nave and the 9:16 for the whole interior were more important than the shape of the aisle itself. Squareness in the vault was less important than whole numbers in the plan.

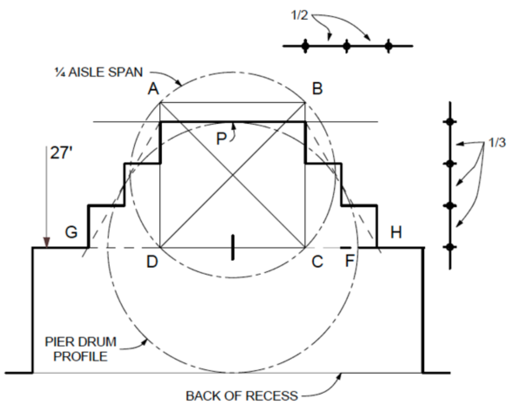
There are many examples of medieval geometry being more important than visual appearance, in contrast to Classical and Renaissance buildings where visual adjustments were important.[14]

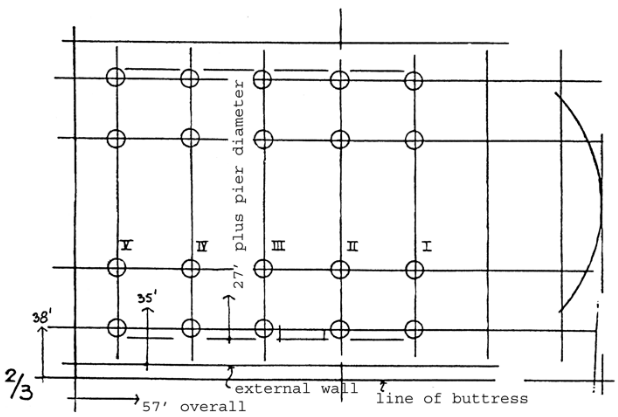
The foot unit

Some have tried to determine foot units by measuring as many items as possible and looking for a common denominator.[15] This will not do, because geometry is a process in which the master may begin with a certain number of feet, but will by successive subdivisions that often involved fraction and irrationals, create the parts from these subdivisions rather than directly from a foot unit.

For example the diameter of the pier is 3/20ths of the span, the base under the torus is 2:√3 of this and the plinth √5:2 of the torus, described in the hexagon above. There is no way the foot measure could be found from these measurements alone. Its natural location lies at the beginning of the design operation, not in the end parts of the geometry, and though it may turn up elsewhere to search for it in the details will only be confusing.

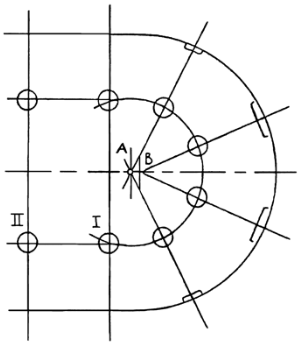
In this case I guessed that the foot unit would be found in the various rectangles. If so, there would be 8 feet in the bay and 14 feet in the span. By calculation from the ideal dimensions the foot unit would measure 363.04 mm. As shown bottom left, the interior space would then measure 27 feet by 48 feet, and the exterior walls were placed 35 feet apart, and the face of the buttresses at 38 feet.

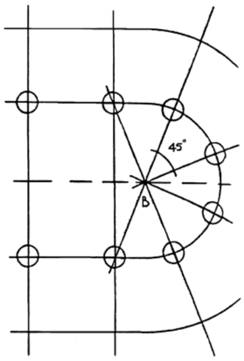


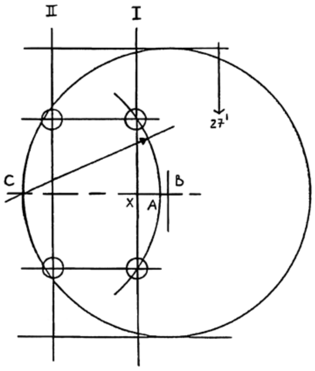


In the other direction the overall length of the chapel from the west face of the western wall to the exterior of the apse measures 57 feet. The building is thus enclosed within a 3:2 rectangle, measuring 57 feet by 38 feet. The interior rectangle was formed from powers of the whole numbers used to create the exterior. The outside walls of the whole tower measure 85 feet by 102 feet, using the same foot as found in the chapel, forming the ratio of 5:6 in units of 17 feet, possibly a rod, pole or perch. The hexagon placed within the side walls locates the axis through X and the first piers of the nave, while the hexagon within that locates the interior of the encasing walls. The western face of the western wall of the chapel Y was placed 42 feet from the east. The analysis of the larger dimensions was made from drawings prepared by the Department of the

Environment. It confirms the foot unit found in the chapel, and suggests that one master was responsible for the entire tower.







Multiple apse centres

Above left shows there are two centres for the arcs locating the hemicycle piers and the wall, and that neither of them coincide with the axis through the first nave piers. This arrangement is too common in medieval buildings to need elaboration.[16]

The curve of the apse walls, inside and out, are set out from B, while the piers are centred on A. This complicates the radials which join the piers to the pilasters. The spaces between the four piers and those between the pilasters are equal, but the central pair of pilasters are twice as wide as the flanking pair. The care taken to ensure that the spaces between the piers and between the pilasters are uniform provides one of the clues to the master's intentions.

In many later buildings the set-out for the apse is much simpler [above middle]. Piers and pilasters are together set out from the one centre B. It is an easy arrangement with equal angles to all the piers, including the first pair in the nave.[17] Yet this arrangement did not appear with the first rib-vaulted apse, but a century later as if the masters had to struggle their way back to this solution against a strong tradition. It seems they may have been too conditioned by the multi-centred approach of their predecessors to see the obvious for some four generations. This suggest that the earlier approach was important, in spite of the manifold difficulties it produced in the vaults.

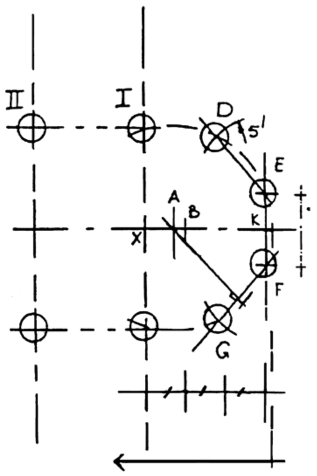
There are a couple of ways in which B may have been located, from which the inner ambulatory wall face was determined from the same 27 feet used in the nave [above right]. The centre A was located by placing the compass on C and drawing the arc through the piers as shown. The distance left between A and X came as quite a surprise, for it turned out to be the same as the diameter of the pier! So, A could have been positioned as the pier diameter from X and then in a sense 'verified' from C.

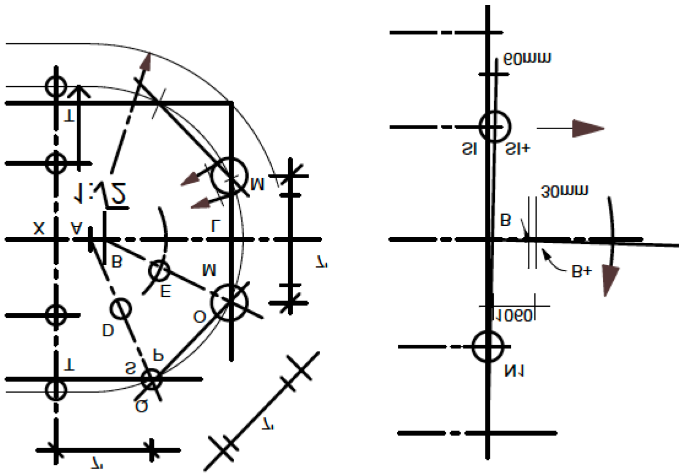
In bottom left the apse piers were set out from the circle drawn from A through the first nave piers.

The piers were then arranged along this arc at 5 foot centres. In bottom right notice how the wall pilasters were also laid out from modules, but instead of there being three equal spaces of 5 feet, the master used a module of 7 feet, or half the span, increased by the pier diameter at the central pilasters, and by half that diameter at the other two.

To summarize, two arcs were used to set out the piers and the pilasters. The piers and pilasters are themselves set out from the modules or foot units employed in the nave. It sounds like playful fantasy, to say the least, except for what happens next.

In bottom right join the pilasters N or M to the centre B. Surprisingly these radials will pass through the piers E and F. Similarly join the other two pilasters Q and R to the centre A, and they too will pass through the appropriate piers D and G! It sounds a little bit like cleverness for its own sake. Clever it certainly is. It took





me days to created a similar apse on my own, and in doing so I came to realize the satisfaction that comes from such an integrated scheme.

Alter the first rectangle of 7:16 in the nave, or the 9:16 to the walls, or any of the other ratios, and you have to start again. It is not easy to do: so why do it?

A single centre B with all the radials determined from it is easier and more suitable for a rib and flying buttress arrangement, as later masters were to find. But then we miss out on the modules. Since that is all we miss out on, their use must have been one of the reasons for using A and B centres, and for accepting the stringent difficulties involved. Modules involve lengths rather than angles, and all are extracted from the nave.

Though I do not understand why these modules should be so important, all that can be said now is that their presence was of the utmost importance, and that much effort was expended in obtaining it.

Circularity

One unusual aspect of the apse is that whichever sequence of geometric steps is used to set it out, we always come back to the beginning. The process is circular. The fact that the axes through the pilasters to the A and B centres meet the piers has just been described as the last step, but in the design process it could have been the first.

Circularity is one of the most common aspects of medieval geometry. I have analysed its occurrence in many examples from Chartres, and have also noticed it in every other monument I have measured. It may be defined as a series of geometric steps involving ratios and figures which are irreconcilable with one another and connected in such a way that one of the last steps will meet up with or repeat an earlier one. As a result it is not possible to say where the geometry began as the first step can be made anywhere.

One example in the nave is where the geometry of the pilaster could have been used to set out the drum of the pier, instead of the other way round. The circularity here is quite splendid. This shows that much effort was expended in every geometry, large and small, to ensure that the process would double back on itself.[18]

Adjustments made to ensure that circularity works

In setting out the apse there was a small discrepancy in the cumulative addition of the modules around the wall. It amounted to 62 mm, previous page right. There was a conflict between and the sequence of wall modules in the apse and the 9:16 geometry in the nave. The master felt compelled to resolve this gap in the dimensions. The way he did this was of the utmost importance, for the same procedure is to be found again and again in medieval buildings. This example illustrates a principle which, through a multitude of individual variations, is one of the more puzzling for us to understand.

He eliminated the discrepancy by adjusting the orientation of the apse. He moved the centre B some 30 mm to the east to B+, twisting the whole apse round the northern pier, which increased the opposite southern by 60 mm, as shown to the right in previous page right. This increased the perimeter of the apse wall, thus eliminating the discrepancy in the modules, while twisting the axis anti-clockwise. All the devices used to locate B were now shifted with the inclination of the axis through the S1 pier. The larger distance XB+ now became exactly 3 feet! A was also moved to A+, and the axis from X+ to L was twisted anti-clockwise by the same 60 mm. However, the distances from X+ to A+ and B+ remain as they were, ensuring that the original ratios would not be lost.

It could be said that one discrepancy was simply replaced by another, but to go to this trouble shows that modular correctness was preferred to the angular. As in the layout of the apse itself correctly maintained modules were preferable to singular centres.[19]

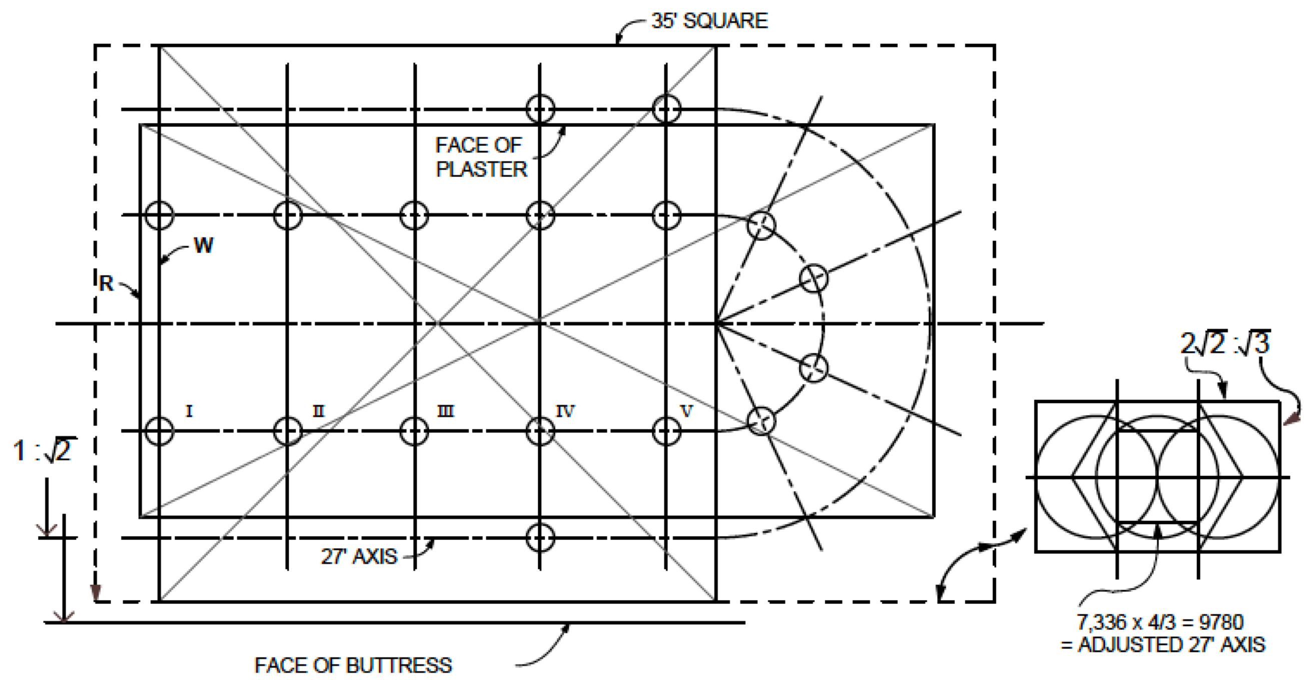
I doubt if these adjustments would have been made during the initial design process. The geometry would have been followed through to its logical end, and then the discrepancies would be used to initiate the adjustments. It is quite extraordinary that in making these adjustments to eliminate the gaps between the systems, it became possible to include a number of additional ratios.

There was an adjustment when the pilasters in the nave were eased inwards at the centre by 45 mm. Doing this introduced another ratio which had long been current in simpler buildings for establishing the thickness of the wall in relation to the interior of the span, that of the square within the circle.

None of these adjustments have removed earlier proportions, for they would still be present longitudinally in the unaltered northern bays, and transversely in the end spans. Yet the adjustments have enriched the geometry, tied up the loose ends, and inserted ideas which were not possible in the original scheme.

Wall thickness

The reason for wall thickness being different in each part of the building has now been answered, for the wall represents the mass between two proportional systems, one determining the interior and the other the exterior. As the systems are appropriate to the place, each part of the building will therefore produce a different wall thickness. This is complicated by another form of circularity, that not only must the wall stem from local systems, but when taken together they must, as the total enclosure of the building, combine to form a geometrically appropriate envelope.



In the above sketch a large 35 feet square determines the external wall-faces in one direction, and in the other it marks the distance from the original B centre to the western interior wall-face.[20] As a result the centres of the western piers are proud of the face of the wall by 19 mm.

In nearly all Romanesque buildings a small gap is to be found where the shaft centre does not lie on the wall face. The reason seems to be invariably the same, that the geometry determining the centre of the shaft is not the same as the one that determined the wall-face.

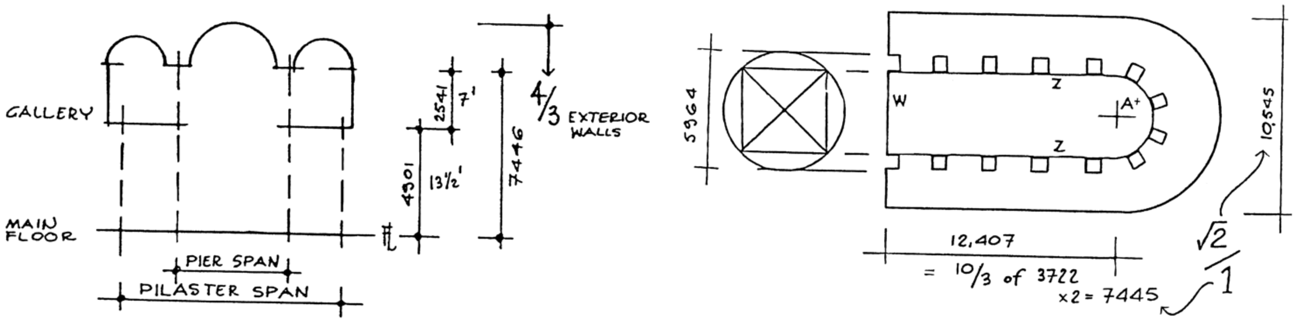
If the wall-face had been located in line with the centre of the pier, the square would not have been present. Conversely if the square had been set out from the adjusted B+ centre the situation would have been reversed so that the pier would have been partly buried within the wall. The geometry was nearly always arranged to expose more than half the pier.

There is a second large unit which has a part to play in locating the wall-faces. It is a double square taken between the faces of the pilasters one way, and in the other from the adjusted interior apse wall to the western wall where it forms the recess between the piers. This is marked R at the western end. This actually exposed another correlation, for now the distance along the longitudinal axis from the eastern piers to R is now exactly one third of a hundred feet, a commonly used medieval dimension. Such an additional pleasure was not possible before the adjustment to the apse B centre.

Thus, the western end, both in the wall-face and the recess within it, reflects through large simple figures both the original and the adjusted positions of the apse. One can be visualised as slipping within the other, the square remaining stationary while the adjustments eased the rectangle eastwards. It would seem that the perfect repetition of ratios and figures was less important than ensuring that as many were incorporated as possible. As long as relationships could be made, then the actual ratios used seem less important than the connection itself.[21]

Extracting the elevation

This medieval phrase is important, for it expresses the techniques used.[22] Dimensions are selected from the plan, and utilised vertically. The transverse space between the pilasters and the span were each halved, and used to locate the gallery floor and the springing of the vaults, measured to the original floor levels.

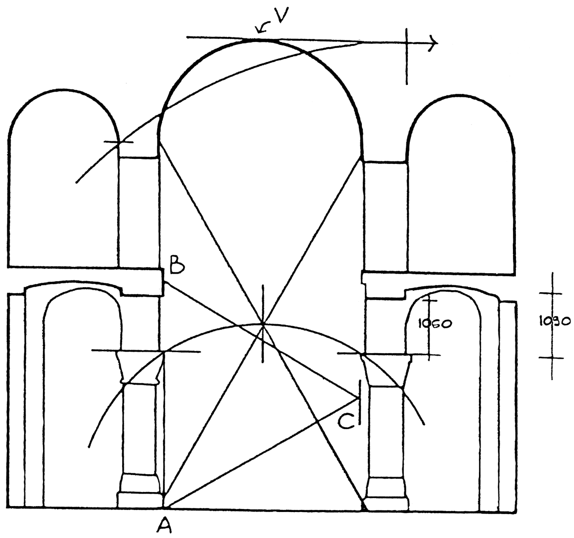


Circularity is present here too, for the addition of these two extracted modules can be formed into a square, the diagonal of which will provide the distance between the wall recesses. This was also made the overall space across the gallery shown above right.

There is also a third example of the combination of hexagons and squares that is not unlike earlier figures, the space between the upper nave walls ZZ relates in its height and width through the hexagon, while the square UV generates the interior space of the gallery and of the double square, plan previous page.

The height of this rectangle is like a summary in the elevation, a sort of highest common denominator, for it not only represents the addition of the span and aisle spaces, but it is also the square within the circle to the overall interior space, and it is the hexagon to the clear internal width of the nave.

It is such a satisfying solution to the different parts that I trust the master got as much pleasure out of producing it as I did. For other steps in extracting the elevations [below] where the heights repeat the essential dimensions of the plan in an orderly sequence, play upon them through a common triangular ratio, and then in the dependent parts like the vaults summarise the larger dimensions of the plan. Thus, is unity maintained.



Summary

All the adjustments made to the plan were deliberate. Be they wall thickness, non-square aisles, apse centres or bent axes, they have a common origin in geometry and stem from four requirements:

- the necessity of closing any gaps between different geometric systems;
- the importance of modules over angles
- and of acceptable ratios over any other factor, including aesthetics; and
- the care with which adjustments were made to enrich the geometry without eliminating any of the ratios already formed.

Unity lay within the geometry rather than within the eye. Through geometry we gain hints of the significance that the numbers and geometric figures may have had for the middle ages. Adjustments not only consolidated the different geometrical aspects, but allowed for the inclusion of other ratios and apparently meaningful lengths which would have not been there otherwise. So, by distorting the perfect symmetry a much richer arrangement was possible. Note how this richness was not overt, but lay hidden within the *process of design*.

Only the wish to ensure that the geometry fitted at each point impelled them to bend and distort what had already been perfected. Where there was a conflict between the regularity of the building and the unity of the geometry, the geometry was always considered the most important. The resulting discrepancies are not errors from carelessness, but deliberate steps in a process that aimed at "one perfect and final solution".

One should also not forget its date, well before the First Crusade, showing that richness in the masters geometry was most probably home-grown and developed among the masons of northern Europe.

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1 Batts "Number symbolism"; Fowler, *Silent poetry*; Hart, "Anglo-saxon art"; Hieatt, "Four feathers"; and Stevick "Mathematical".

2 Fuller "French style"; Newmann "Counter-reformation" and "External evidence"; Pont, "Revolution".

3 Albarn, *et al*, *Language of pattern*; Bain, *Celtic art*; and Swenson, "Symmetry potentials".

4 Grove, *Dictionary*. 27.

5 For example Bain, *Celtic art*, 105; Crosby, "Western bays", 41; and van der Meulen, "Recent literature", 163.

6 Anthony, *French baroque*, 368.

7 Stevick, "Mathematical". Stevick wrote "textual defects or irregularities" and "confused by a copyist who consequently omitted the line", "Geometric design", 103.

8 McClain, *Invariance*; and McClain "Pythagorean Plato" on *tuning* as the development of regular musical modules based on whole numbers, and *tempering* as empirically adjusted proportions which are irrational.

9 Pont, "Revolution". His work is supported by two other Australian musicians, Dene Barnet of Flinders University who is working on Haydn"s irregular phrasing, and Dennis Vaughan from the State Opera of South Australia on Verdi and Puccini, Vaughan, "Inner language".

10 Tovey, *Beethoven*, 7.

11 James, *Contractors;* James, *Chartres*; and Shelby, "Chartres", 173-78.

12 Brown, "Tower of London"; Charlton, *Buildings*; Colvin, *Kings works*; and Peers, "Development", who dates the start of construction in 1087 or before, and its completion some time before 1100.

13 "26 Measuring and extracting" .

14 Published examples are Bannister, "St Peter"; Conant, *Cluny*, and James, *Contractors*, ch. 8.

15 Sunderland, "Symbolic numbers"; Wu, "Hughes Libergier".

16 Branner, "Cathedral of Reims"; and Crosby, "Crypt and choir" shows similar discrepancies as here.

17 Branner, "Cathedral of Reims".

18 "26 Measuring and extracting"

19 Bent axes at Chartres, James, *Contractors*, 142-43.

20 The external wall-face was established in Fig. 32-5.

21 Seldom does one find a western wall placed square to the longitudinal axis. At Chartres the inclination of this wall initiated the 150 mm twist in the transverse axes of the nave, and permitted the master to incorporate the numbers of the Easter Cycle along the length of the cathedral – James, *Contractors*, 143 and 155. At Cluny it permitted the series of Perfect Numbers to be incorporated down one side of the building and 601 as the sum of Alpha and Omega down the other: Conant, *Cluny*, n. 17; and Conant, "Excavations at Cluny".

22 Shelby, *Gothic design*, n.11 and Ackerman "Cathedral of Milan".